Making Algebraic Connections 4-8 (MAC)

Facilitated by: Partnership Institute for Math and Science Education Reform (PIMSER)

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PIMSER
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Overview of the MAC Grant

The Making Algebraic Connections project (MAC) provided the content and strategies necessary for grade 4-8 teachers to engineer effective classroom experiences that help students self-assess and confront their misconceptions in algebraic reasoning. MAC content focused on deepening participating teachers' understanding of the content and pedagogical content knowledge necessary to teach the algebra in grades 4-8. The project addressed the foundational prerequisites that research identified as necessary for success in algebra along with the learning progression in algebra from grades 4-8 as outlined in the Kentucky Academic Standards (KAS) for mathematics and the connection to the Standards for Mathematical Practice (SMP) necessary for students to develop conceptual understanding.

MAC content included the use of innovative technology applications and a variety of instructional tools and strategies that support student learning. This content allowed teachers to learn how to provide effective Tier 1 intervention including differentiated learning experiences. Also, teachers explored instructional tools and strategies that helped them to provide learning experiences for students to develop the prerequisite conceptual knowledge needed to be successful in algebra in high school and beyond through vertical conversations and activities highlighting the progression of algebraic thought from intermediate to middle school. The project developed teacher capacity to gather evidence of student progress towards the Standards and to produce a bank of strategies and resources teachers can utilize when students need additional support to meet the Standards.

There were many instructional strategies that were learned with teacher-generated products. Two such products highlighted in this report are Task Rotations and Assessment Menus. Overviews of each instructional strategy are to follow with teacher generated products and teacher notes to guide others in the implementation of these products in their classrooms.

Overview of Task Rotations

Task rotations are an instructional strategy that can be used to formatively assess students' understanding of a particular mathematical topic. Based on the work of Harvey Silver and his associates at Thoughtful Classroom, a task rotation presents four distinct tasks that represent each of the four learning styles (Mastery, Understanding, Interpersonal and Self-Expressive).

Task rotations push students to reason and analyze their understanding of a mathematical topic in different styles. Implementation varies among teachers. Some encourage students to try all the tasks while other teachers provide a student-choice option. Regardless of how task rotations are implemented, the four tasks provide an opportunity for differentiation of instruction.

Teachers in the MAC grant received extensive training in Harvey Silver's Math Tools before creating their own task rotation. MAC teachers paired their knowledge of KAS and along with their work on the SMPs in workshops in order to develop the mathematical framework needed to write the task rotation. MAC teachers completed a book study and learned about the variety of instructional tools used by each of the four learning styles. The task rotations along with Teacher Notes to help guide implementation are included in the following section:

Teacher Notes for Task Rotation

4.NBT.3 Place Value

Rationale for Task Rotation:

This task rotation centers around the idea of rounding whole numbers up to the nearest hundred and thousand. Rounding serves as a critical skill as students develop their estimating skills for determining the reasonableness of an answer. The use of models, diagrams and number lines is critical in the laying the conceptional development of this standard for student understanding. This task rotation allows students to describe the process of rounding in their own terms and then using this knowledge to round numbers in a variety of mathematical and real-world examples.

CCSS-M Standard(s):

4.NBT.A.3. Use place value understanding to round multi-digit whole numbers to any place.

Standards for Mathematical Practice (SMP):

The tasks could utilize many of the SMPs. Below are the major SMPs that each task utilizes.

Mastery Task:

SMP 6: Attend to precision.

Students identify the individual procedural steps on their way to solving a problem using rounding.

Understanding Task:

SMP. 4: Model with Mathematics

Students illustrate using a model such as a number line or diagram to justify how they rounded a number to the nearest hundred or thousand.

Interpersonal Task:

SMP 3: Construct viable arguments and critique the reasoning of others.

Students engage in deep forms of thinking by examining a situation closely and applying mathematical concepts and procedures to determine who's right.

SMP 6: Attend to Precision

Students examine claims and check for errors. Also, students express numerical answers with appropriate degree of precision and carefully formulated explanations.

Self-Expressive Task:

SMP 2: Reason Abstractly and Quantitatively

Students reason (decontextualize) how quantities they see around them in real-life could be mathematically represented (contextualized).

Misconceptions:

When rounding students do not understand the convention for rounding because they often do not understand place value. For example, when rounding 12, 516, students struggle finding the digit in the thousands place. This misconception is perpetuated when teachers use rhymes that are not grounded in conceptual understanding for teaching students how to round numbers.

For example,

Round 147,589 to the nearest ten-thousand:

a. Find the place

b. Look next door

c. 5 or bigger...Add 1 more!
d. 4 or less… Let it rest!

While it might seem like a great trick to get students to round, students misconstrue the poem, thus perpetuating the misconception.

Additionally, if teachers decide to use number lines to demonstrate how to round (helping to bolster conceptual understanding), students' lack of understanding of place value can impact students' ability to construct and understand the meaning behind the number line.

Implementation of the Task Rotation:

This task rotation offers an additional formative assessment for teachers to incorporate in their classrooms. This rotation can be assigned as bell work, class work, or homework. Upon first exposure to a task rotation, students will need a great amount of guidance in what is expected of each task. Completing just one of the tasks at a time builds a knowledge of what will be expected of them in the future. There is also a great deal of leeway in how they are implemented. Student may select two or three to complete or a teacher may assign all four tasks. These tasks are not to be used as work left for a substitute in the absence of the teacher.

Task Rotation: 4.NBT.3 Place Value

Standard: 4.NBT.3: Using place value understanding to round multi-digit whole numbers to any place.

Mastery	**Interpersonal**
Create a fist list of steps you feel you need in order to round numbers to the nearest hundred or thousand.	Big Creek Elementary needs to purchase water bottles for field day. If 1,529 students attend the school, about how many water bottles should be purchased if the water comes in cases of 100? Mary says to round to the nearest hundred since water comes in cases of 100. Joe says if rounded to the nearest hundred every student won't get a water bottle. Who is right? Explain your reasoning.
Understanding	**Self-Expressive**
Round the following numbers to the place specified: • 1,529 rounded to the nearest hundred. • 1,529 rounded to the nearest thousand. Prove your answer by representing with models (example…number lines, diagrams).	Create a real-life scenario in which the following mathematics is modeled: a) A situation in which the answer when rounded to the nearest hundred will be equal to 1200. b) A situation in which the answer when rounded to the nearest thousand will be equal to 3000.

Teacher Notes for Task Rotation:

6.NS.B.3 Operations with Decimals

Rationale for Task Rotation:

This task rotation focuses on using the standard algorithm of operating with decimals using all four operations. At this level students are expected to fluently use the standard algorithm to add, subtract, multiply, and divide with multi-digit decimals. Standard algorithm first appears in the CCSS in 3rd grade (add and subtract using whole numbers); in the 4th grade (add and subtract with the standard algorithm using multi-digit whole numbers) and in 5th grade (modeling decimals, place value and properties). This task rotation provides students opportunities to examine the standard algorithms and practice its use with a variety of tasks. Students will hone their understanding of the use of standard algorithms by: deriving patterns seen in various problems, writing how they are used in real-world scenarios and examining others' reasoning for error analysis.

CCSS-M Standard(s):

Standard: **6.NS.B.3** - Fluently add, subtract, multiply, and divide multi-digit decimals using standard algorithm for each operation.

Standards for Mathematical Practice (SMP):

The tasks could utilize many of the SMPs. Below are the major SMPs that each task utilizes.

Mastery Task:

SMP 6: Attend to Precision.

Students write definitions and use representations to make connections among the terms. Students learn to communicate math ideas using appropriate terminology through the use of this vocabulary.

Understanding Task:

SMP 2: Reasoning Abstractly and Quantitatively.

Students contextualize the example problems to determine how the placement of the decimal point is derived by the use of the standard algorithm. Students search for patterns within the quantities provided and how the units (whole numbers and decimal parts) are represented.

Interpersonal Task:

SMP 3: Construct a Viable Argument and Critique the Reasoning of Others.

Students argue for or against a stance based on their knowledge of standard algorithms and justify their thinking with examples.

Self-Expressive Task:

SMP 4: Model with Mathematics.

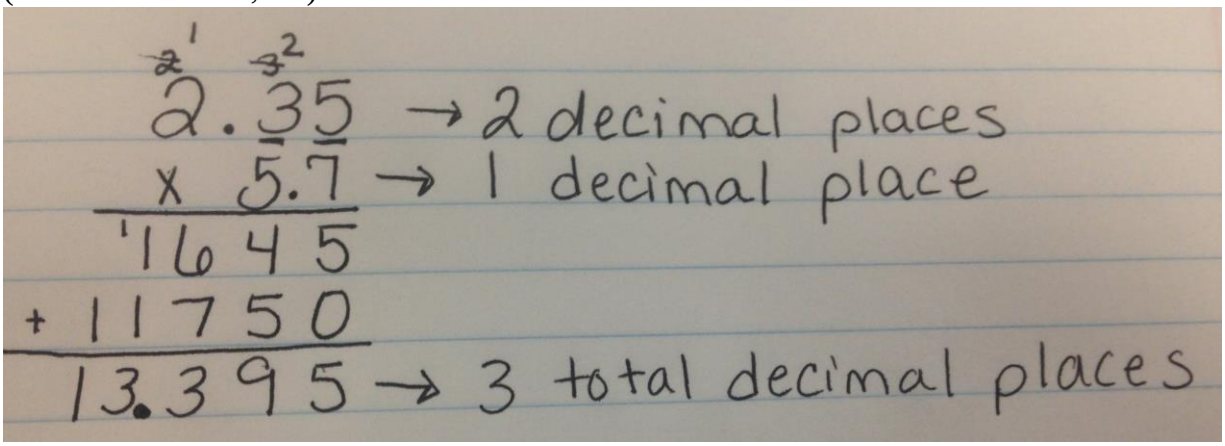
Students create and model a real-world situation where they are use the appropriate operations to solve a decimal operation problem.

Misconceptions:

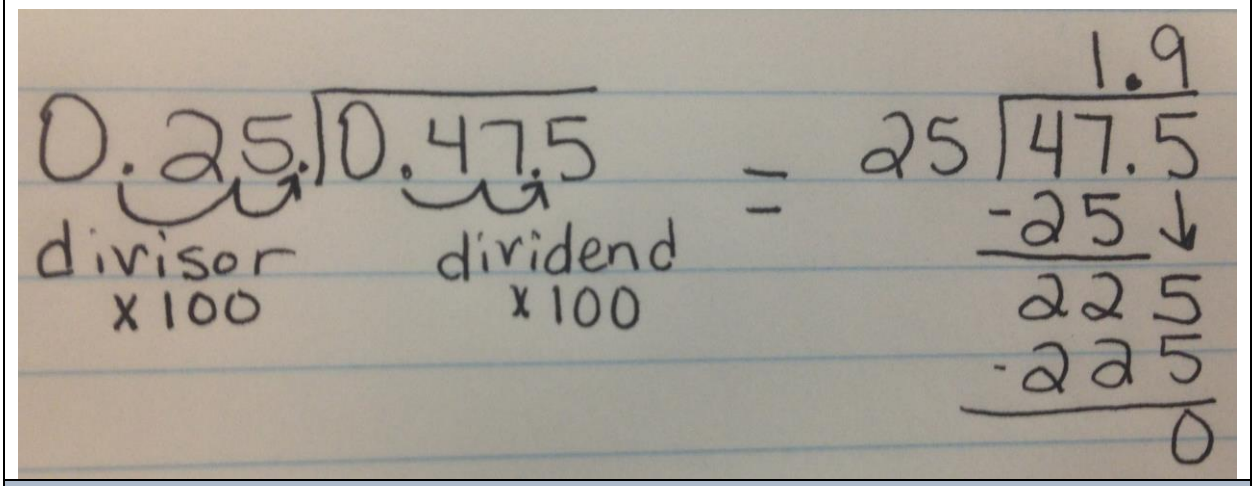
Students often confuse how to use the decimal when adding, subtracting, multiplying, and dividing. This task pushes students to use and support their reasoning effectively.

It holds true that when adding and subtracting with decimals that the "decimals line up," in turn you are lining the values of each digit according to place value. When using the standard algorithm the decimal would come "straight down" to maintain its correct placement between the whole number and decimal parts.

To multiply with decimals, the decimals do not have to be "lined up." You will multiply as with whole numbers. Once the product is achieved, students will refer back to the multipliers to determine the decimal value of the product. The total number of decimal digits in the multipliers will be the number of decimal places in the product. Hence, if you are multiplying a number in the hundredths by a number in the tenths your product will be in the thousandths. (1/100 x 1/10 = 1/1,000).



When dividing with decimals the divisor must be a whole number. To change a decimal number into a whole number, move the decimal to the end of the number. Mathematically you are multiplying by a power of 10. If the divisor has been multiplied to achieve a whole number, the dividend must be multiplied by the same power of 10. (You can relate this back to equivalent fractions.) The decimal in the dividend will go straight up into the quotient.



Implementation of the Task Rotation:

This task rotation is an excellent way in which to formatively assess your students. These tasks can be assigned as bell work, class work, or homework. It is important that these tasks are not to be assigned at one time. (Students could choose one task to show their understanding of this standard. These tasks are not to be used for work that a substitute teacher assigns in a teacher's absence.

Task Rotation: 6.NS.B.3 Operations with Decimals

Standard: 6.NS.B.3- Fluently add, subtract, multiply, and divide multi-digit decimals using standard algorithm for each operation.

<table>
<tr>
<td>Mastery

Complete the knowledge cards for each term on the list below. Include an icon, drawing, or graph that illustrates the term. Create a word web to show how the terms relate to each other.

Decimal addition
Decimal subtraction
Decimal multiplication
Decimal division
Standard Algorithm</td>
<td>Interpersonal

James and Jon are learning about operations with decimals. James says that the decimals always need to be lined up no matter what the operation. Jon disagrees and says that is not the case.
Who is correct and why? Provide examples to justify your reasoning.</td>
</tr>
<tr>
<td>Understanding

Examine the following problems. Describe any patterns in the decimals you notice: a) between the problem given and the answer and b) among the different problems.

Problem A
25.3
+ 7.21
32.51

Problem B
16.8
-10.9
5.9

Problem C
9.3
X 2.45
22.785

Problem D
$9.72 \div 3.6 = 2.7$</td>
<td>Self-Expressive

Create your own real-world scenario where at least one decimal operation would be used. Describe the situation and solve your problem using the standard algorithm for each operation.</td>
</tr>
</table>

Teacher Notes for Task Rotation

7.NS.1: Addition of Integers

Rationale for Task Rotation:

The rationale for this task rotation is two-fold. First, students often do not learn about how a mathematical topic is found in the world around them. Students will work on real-world representations and scenarios that illustrate addition of integers. Second, students will learn about common misconceptions associated with the addition of integers and how to use error analysis to debunk these misconceptions.

CCSS-M Standard(s):

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Standards for Mathematical Practice (SMP):

The tasks could utilize many of the SMPs. Below are the major SMPs that each task utilizes.

Mastery Task:

SMP 3: Construct viable arguments and critique the reasoning of others

Students must use their knowledge of signs rules and integers to determine how to correct problem and justify their reasoning.

Understanding Task:

SMP 6: Attend to precision

Students try to communicate precisely to others whether a mathematical statement is true always, sometimes or never with use of counterexamples, correct application of sign rules and vocabulary.

Interpersonal Task:

SMP 2: Reason abstractly and quantitatively

Students must apply the mathematics they know to represent real-world situations. They must be able to identify important quantities around them and represent using correct mathematical symbols.

Self-Expressive Task:

SMP 4: Model with mathematics

Students are asked to represent mathematical quantities in a variety of models.

SMP #5 Use appropriate tools strategically

Students are given a variety of models they can use to represent a quantity. No particular model was specified for students to use.

Misconceptions:

Operations with signed numbers is problematic for many students. Quite often, teachers state a variety of sign rules and do not take the time to conceptually build students' understanding of integer operations. Quick sign rules such as:

- if the signs are the same, then the answer is positive or
- if they are different, they are negative

may be partially correct for multiplication and division of integers but do not necessary translate to addition of integers. Students confuse these quick sayings without thinking about the application of them to specific operations such as addition. This task rotation pushes

students to examine a variety of models and representations along with error analysis to build more of a conceptual understanding of integer addition,
Implementation of the Task Rotation:
This task rotation is an excellent way in which to formatively assess your students. These tasks can be assigned as bell work, class work, or homework. It is important that these tasks are not to be assigned at one time. (Students could choose one task to show their understanding of this standard). These tasks are not to be used for work that a substitute teacher assigns in a teacher's absence.

Task Rotation

7.NS.1: Addition of Integers

Standard: 7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Mastery **What's wrong?** Directions: Correct the signs, as needed, of the integers in the following problems to get the given sums. Justify your solution. 7 + (-12) = 5 3 + (-4) = -1 (-6) + 9 = -3	**Interpersonal** **Where in the world?** Directions: Write AND solve a real-world problem based on each set of given criteria. ● One problem that includes the **sum** of two positive integers ● One problem that includes the **sum** of two negative integers ● Two problems that include the **sum** of a positive and a negative integer
Understanding **Sometimes-Always-Never** Directions: Determine whether each statement is true or false. If the statement is false, provide a counterexample to show it is false and then change the underlined word to make it a true statement. ● The sum of a negative integer and a positive integer is <u>always</u>negative. ● The sum of two negative integers is <u>sometimes</u> negative. ● The sum of two positive integers is <u>never</u> negative.	**Self-Expressive** **Modeling with Manipulatives** Directions: Using a number line, algebra tiles. or two-color counters, model the solutions to the following problems. Explain your answers. -5 + 3= 3 + (-6)=

Teacher Notes for Task Rotation:

7.NS.1 - Add/Subtract Integers

Rationale for Task Rotation:

The reason for this task rotation is to extend and ensure that students understand the processes of adding and subtracting integers. The mastery task assumes students understand the process of taking averages and uses negative integers to push students' understanding of adding and subtracting integers. The next task, interpersonal, lets the students reflect and express what they understand about addition and subtraction of integers and provides a forum for discussion. The next task, understanding, presents the students with a variety of questions where they will demonstrate their knowledge of adding and subtracting integers. The final task, self-expressive, allows the students a chance to model the process of adding and subtracting integers by writing a number sentence and modeling through the use of vertical and horizontal number lines. All of these tasks work together to help students move forward in their understanding of addition and subtraction of integers.

CCSS-M Standard(s):

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Standards for Mathematical Practice (SMP):

The tasks could utilize many of the SMPs. Below are the major SMPs that each task utilizes.

Mastery Task:

SMP 2: Reason Abstractly and Quantitatively

Students must make sense of the quantities presented in order to find the average. They can be flexible with their knowledge of the properties of operations to allow them to add the integers.

Understanding Task:

SMP 3: Construct Viable Arguments and Critique the Reasoning of Others

As students state whether they agree or disagree with the given statements they are also required to explain their solution. To coincide with SMP 1 students are also deciding if the statements make sense in relation to integers.

Interpersonal Task:

SMP 2 Reason Abstractly and Quantitatively

This task provides an open approach for students to show their understanding of integers. The easy to use format makes it easier for students to create a coherent response.

SMP 6: Attend to Precision

As students are responding to the prompt it is crucial that they communicate effectively. They will use precise terminology to support their explanations.

Self-Expressive Task:

SMP 4: Model with Mathematics

This task requires students to create a unique number sentence using integers as well as creating a model to show their number sentence. They must work with a partner which will provide immediate feedback. Models will help students make sense of their problems and make changes as necessary.

Misconceptions:
Students often make the mistake of using multiplication and division rules when solving addition and subtraction problems. It is important to give students time to reason about the numbers presented and the size of the positive and negative numbers before introducing the rules. Students also have trouble separating whole numbers from integers. In a problem such as 10-(-7), students think they can subtract 7 and the answer is 3. They are not acknowledging the size of -7.
Implementation of the Task Rotation:
This task rotation is an excellent way in which to formatively assess your students. These tasks can be assigned as bell work, class work, or homework. It is important that these tasks are not to be assigned at one time. (Students could choose one task to show their understanding of this standard). These tasks are not to be used for work that a substitute teacher assigns in a teacher's absence.

Task Rotation: 7.NS.1 - Add/Subtract Integers

Standard: **7.NS.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Mastery **What's Wrong?**	**Interpersonal** **I know what I know about…**
The Daily Temperatures this week were: 4 13 -2 -6 -1 -10 4 James says: Average is 5.7 Jane says: Average is 3.5	Fill out the following based on what you've learned about integers. I know what I know about… First, I know… In addition, I Know… Finally, I Know… Now you know what I know about….
Understanding **Support or Refute**	**Self- Expressive** **Modeling Manipulation**
Look at these 4 statements, decide if you support (agree) or Refute (disagree) with the statement and tell why. 1. You can't subtract negative numbers. 2. Subtracting is the same as adding a negative. 3. -10 degrees is colder than 0 degrees. A negative added to a positive will always give you a negative answer.	Write two math sentences using integers. One sentence should model addition while the second should model subtraction. In each math sentence use at least one negative integer. Model and solve the sentences using horizontal or vertical number lines.

Overview of Assessment Menu

The assessment menu is an extension of the task rotation. The four learning styles (Mastery, Understanding, Interpersonal and Self-Expressive) that are a part of Harvey Silver's work are the basis for this instructional strategy. Assessment menu debunks the idea that one form of assessment fits all students needs.

For the assessment menu, a teacher creates three levels of task for each learning style. The levels (1, 2, and 3) differ in the cognitive demand required by the student to answer the task. Depending on the learning style, the levels move in complexity of student thought from defining, listing, comparing/contrasting, express opinions (Level 1) to creating diagrams, explanations, speculations and reflection (Level 2) to creating a variety of representations, creative products (art, poetry, inventions) and defend positions (Level 3).

Student-choice is key in this instructional strategy. It is not the goal for a student to complete all twelve tasks, but rather choose four tasks. They are to choose one from each learning style and one from each level of complexity. Student-choice allows for this strategy to easily differentiate assessment. It is important to note that some tasks within the assessment menu may need scoring rubrics with success criteria listed in order to properly assess students' work.

The assessment menus with guiding teacher notes for implementation are included in the following section.

Teacher Notes for Assessment Menu:

4.MD.3 Area and Perimeter

Rationale for Assessment Menu:

This assessment menu focuses on the concepts relate to area and perimeter. Area and perimeter are two important and fundamental parts of geometry which provide context for many real-world problems. This assessment menu provides multiple opportunities for student to showcase their knowledge of what is area and perimeter along with comparing and contrasting these important geometric topics.

Assessment Menu Levels of Complexity Explained:

The assessment menu utilizes the four learning styles of mastery, understanding, interpersonal and self-expressive. Each learning style has three levels of complexity:

Level One:
Students will define vocabulary, agree or disagree, create a rectangle, and prioritize knowledge.

Level Two:
Students will create diagram, support with evidence, speculate all possible outcomes, and reflect through writing.

Level Three:
Students will demonstrate knowledge, propose solutions, invent a solution, and act on feelings.

CCSS-M Standard(s):

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

Standards for Mathematical Practice (SMP):

The tasks could utilize many of the SMPs. Below are the major SMPs that each task utilizes.

Mastery Task
SMP 1: *Make sense of problems and persevere in solving them*.
Students will define area and perimeter and label length, width and area. They will use the length and width to calculate both area and perimeter.

Understanding Tasks:
SMP 2: Reason abstractly and quantitatively.
Students will make sense of the problem to determine the correct area and perimeter.

Interpersonal Tasks:
SMP 7: Look for and make use of structure.
Students will use knowledge of properties to efficiently solve problems (find area and perimeter).

Self-Expressive Tasks:
SMP 8: Look for and express regularity in repeated reasoning.
Students will evaluate the reasonableness of their answers and make generalizations based on findings.

Misconceptions:

There are two major misconceptions of perimeter and area. First, students interchange the definitions of perimeter and area. Quite often these concepts are taught by introducing the

formula first rather than having students build a conceptual understanding of the two concepts. Secondly, students confuse the formulas for area and perimeter. Perhaps this is because they confuse the idea of perimeter and area. Teachers also need to be aware of their language when instructing about area of rectangles. Often teachers create another misconception by noting that the area formula for rectangles is the only area formula. This is only one of many area formulas used for polygons. Consistent reminders and spiral review are excellent ways to counteract this misconception. Constant exposure to real life example is critical.

Implementation of the Assessment Menu:

This assessment menu is an excellent way in which to formatively assess your students. These assessment tasks could also be summative. (Note: Some tasks require a scoring guide or rubric for proper summative evaluation). However, teachers need to note that not all tasks are assigned for students to complete. **Students are to select at least one task from each row (complexity level) and one task from each column (learning style).** The purpose of these tasks is to push student thought in a variety of modalities. Also, student choice is key with an assessment menu. Differentiated instruction and assessment is a key outcome of implementing the assessment menu. These tasks can be assigned as bell work, class work or homework. It is important that these tasks are not to be assigned at one time. These tasks are not to be used for work that a substitute teacher assigns in a teacher's absence.

Assessment Menu: 4.MD.A.3 Area and Perimeter

Standard: 4.MD.A.3: Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

Level	Mastery	Understanding	Self-Expressive	Interpersonal
1	Create a chart for 4 vocabulary words you will need to know in order to understand perimeter and area. Word | Definition | Visual Picture	Write "A" if you agree with the statement and "D" if you disagree. Show work to prove your answer. 8 ft. 2 ft. 1. The area is 10 ft^2 2. The area is 16 ft^2 3. The perimeter is 20 ft 4. The perimeter is 10 ft	Create a rectangle for which the perimeter is twice the area.	Copy the following organizer and fill in the blanks with information about your knowledge of area and perimeter. I know what I know about area and perimeter. First, I know _____ In addition, I know _____ Finally, I know _____
2	Create a rectangle • Use yellow to show the length • Use black to show the width • Use blue to show the area • Use red to show the perimeter	If the area of the rectangle is 112 cm^2, which of the following perimeters would NOT be correct? A. 44 cm B. 48 cm C. 64 cm D. 116 cm	List as many dimensions of a quadrilateral that will result in an area of 24 yds^2	Jonah is buying fencing for his dog lot. If the fence cost $2 a foot which option would be the least expensive? A. Fence to enclose an area of 24 ft^2 B. Fence to enclose an area of 16 ft^2 Justify your reasoning.
3	Find the width and perimeter of the figure below: 8 in w 32 in^2	If the length of the rectangle is 7 feet and the width is twice the length, what is the area?	The area of Kendall's garden is 450 feet square. The garden is 9 feet wide. How many feet of fencing will Kendall need to buy to enclose the garden on all four sides?	What else would you like to add to demonstrate your knowledge and understanding of area and perimeter?

Teacher Notes for Assessment Menu:

5.OA.1 & 2: Order of Operations and Expressions

Rationale for Assessment Menu:

This assessment menu focuses on the idea of creating and evaluating numerical expressions using the order of operations. It also focuses on student understanding of how to interpret mathematical symbols and apply the order of operations to simplify expressions. (To what extent do they determine which step to take in first evaluating the expression or offer an explanation that reflects understanding of the order of operations?) The CCSS-M denotes students are "understanding and explaining the steps in the order of operations, the purposes of parentheses, brackets, or braces in numerical expressions, and the difference between numerical expressions and numerical equations". The skill of creating and evaluating numerical expressions is foundational learning in preparation for the Expressions and Equations domain in grades 6–8.

Assessment Menu Levels of Complexity Explained:

The assessment menu utilizes the four learning styles of mastery, understanding, interpersonal and self-expressive. Each learning style has three levels of complexity:

Level One:
Students will list, compare/contrast, think divergently, and prioritize thinking of numerical expressions and the order of operations.

Level Two:
Students will sequence, support/refute with evidence, transform ideas, and reflect through writing.

Level Three:
Students will demonstrate the procedure of evaluating numerical expressions, debate ideas, design a creative product, and act on feelings.

CCSS-M Standard(s):

5. OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5. OA. 2 Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product*.

Standards for Mathematical Practice (SMP):

The tasks could utilize many of the SMPs. Below are the major SMPs that each task utilizes.

Mastery Task:
SMP 1: Make Sense of Problems and Persevere in Solving Them
Students demonstrate a baseline understanding of evaluating expressions using the order of operations. Problems and tasks are presented to students in a variety of ways.

Understanding Tasks:
SMP 3: Construct viable arguments and critique the reasoning of others
Students will agree or disagree of presented problems using acquired knowledge throughout the unit of study.
SMP 7: Look for and make use of structure.

Using knowledge of order of operations students will efficiently structure the expressions beginning with the original expression, then solve the problem, and the logical order of working from the expression to the answer.

Interpersonal Tasks:
SMP 1: Make Sense of Problems and Persevere in Solving Them
Students understand and make sense of the concept of equivalent expressions. Students will need to be able to persevere to solve numerical expressions and match up the equivalent expressions.

Self-Expressive Tasks:
SMP 6: Attend to Precision
Students will use topics such as their ages and favorite numbers to generate numerical expressions. Students will have to use specific vocabulary to create a poem or song to demonstrate knowledge of the order of operations.

Misconceptions:

This task rotation is designed to aid students in overcoming common misconceptions in working with numerical expressions. Students often evaluate an expression moving left to right, with no consideration of the operations. For example: 32 ÷ 4 + 27 x 3. Note: If a student didn't understand the left to right concept of multiply and divide, he/she might try to solve the problem going from left to right. Example: A student is given 32 ÷ 4 + 27 x 3. The student may multiply the 27 and 3 before dividing the 32 and 8. That is incorrect. Student should divide first, followed by multiply, and finish with addition. Others will look for the "easy" parts to solve first. Building an understanding that there is a specific ORDER in place is invaluable for students as they move into more difficult algebra standards in the middle grades. Additional focus needs to be placed on the left to right evaluation of multiplication/division and addition/subtraction. Students become locked into the PEMDAS order and frequently forget that the M and D, along with A and S, work LEFT TO RIGHT. For example, 32 ÷ 4 x 27 + 3. Students may want to multiply 4 x 27 in the middle before completing the division on the left side.

Implementation of the Assessment Menu:

This assessment menu is an excellent way in which to formatively assess your students. These assessment tasks could also be summative. (Note: Some tasks require a scoring guide or rubric for proper summative evaluation). However, teachers need to note that not all tasks are assigned for students to complete. **Students are to select at least one task from each row (complexity level) and one task from each column (learning style).** The purpose of these tasks is to push student thought in a variety of modalities. Also, student choice is key with an assessment menu. Differentiated instruction and assessment is a key outcome of implementing the assessment menu. These tasks can be assigned as bell work, class work or homework. It is important that these tasks are not to be assigned at one time. These tasks are not to be used for work that a substitute teacher assigns in a teacher's absence.

Assessment Menu

5.OA.1 and 5.OA.2 Order of Operations and Expressions

Level	Mastery	Understanding	Self-Expressive	Interpersonal
1	Create a "fist list" for the order of operations.	Examine the options below. Read and carefully consider each of the following statements. Place an "A" next to the mathematical statement if you agree with the answer. Place a "D" next to the mathematical statement you disagree with AND correct it. 1. 3 + 7 x 2 = 20 2. (12 + 3) – 9 + 4 = 10 3. $12 + 15 \div 3 = 9$	What might the expression be to equal your current age. Make sure to include one of the following: Parentheses, brackets, or braces. Also use at least two of the four operations.	Copy the following organizer and fill in the blanks with information about your knowledge of the order of operations. I know what I know about the order of operations. First, I know _____ In addition, I know _____ Finally, I know _____
2	(24 x 80) + 24 x 4 -2 Identify the order of steps to solve the expression above. 1. 2. 3. 4.	Alex solved the following equation: 22-7 x (5-3)= He used the steps below: 22 - 7 x (5-3) 22 - 7 x 2 15 x 2 30 Do you agree or disagree with Alex. Support your answer.	Choose your favorite number between 50-100. Create as many expressions as possible that will equal that number. Create at least 10 expression, 5 of which must include parentheses, brackets, or braces. Make sure to use the four operations throughout the 10 expressions.	Order of operations can contain many different steps and components such as parentheses, brackets, braces, symbols, exponents, etc. What do you feel is easy for you when doing the order of operations and why do you find it easy? What is difficult for you when doing the order of operations and why do you find it difficult?
3	Demonstrate how to solve the following by writing each step for the following problem: 8 x {16- (23-17)} +5 1. 2. 3. 4.	Sara and Max both solved the equation below. 5 + 4 x (10-8) Sara says the answer is 18. Max says the answer is 13. Determine who is correct and why? What mistake did the other person make?	Create a song or write a poem that would be helpful for others to know the steps of order of operations, and the importance of those steps.	What else would you like to add to demonstrate your knowledge and understanding of order of operations?

Teacher Notes for Assessment Menu:

7. RP.A.2 Proportional vs. Non-Proportional Relationships

Rationale for Assessment Menu:

This assessment focuses on idea of comparing not only the difference between proportional and non-proportional relationships, but also how each can be modeled in a variety of representations (graph, table, scenarios and equations). The CCSS-M denotes four critical areas of instructional focus in seventh grade. One of these areas is proportional reasoning, which CCSS-M states students are "developing understanding of and applying proportional relationships". Not only do the CCSS-M require students to think proportionally about percentages and scale drawings, but it states that students must "distinguish proportional relationships from other relationships" along with students should "graph proportional relationships and understand unit rate informally as the measure of the steepness of the related line, called the slope". The skills of interpreting and representing proportional relationships is foundational learning for future of learning of slope and rate of change which is bedrock of algebraic reasoning found in eighth grade and high school mathematics.

Assessment Menu Levels of Complexity Explained:

The assessment menu utilizes the four learning styles of mastery, understanding, interpersonal and self-expressive. Each learning style has three levels of complexity:

Level One:
Assessment tasks are utilized for students to organize, list and represent characteristics of proportional and non-proportional relationships along with their representations.

Level Two:
Assessment tasks push students to analyze, reason, explain, classify and compare/contrast characteristics of proportional and non-proportional relationships along with their representations.

Level Three:
Assessment tasks provide an opportunity for students to analyze how the characteristics of proportional and non-proportional relationships and representations are seen around them in everyday life. Level Three assessment tasks also push students to create their own representations and graphic organizers to demonstrate how they understand the key characteristics of proportional and non-proportional relationships and representations.

CCSS-M Standard(s):

7. RP.A.2: Recognize and represent proportional relationships between quantities.

7. RP.A.2.A: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

7. RP.A.2.B: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Standards for Mathematical Practice (SMP):

The tasks could utilize many of the SMPs. Below are the major SMPs that each task utilizes.
Mastery Tasks:
SMP 4: Model with Mathematics:

The progression of the mastery tasks pushes students to move from listing characteristics of proportional and non-proportional representing them to finding misconceptions in a variety of models.
SMP 6: Attend to Precision:
Students practice using their vocabulary skills in their explanations for these tasks. They must also examine a variety of representations to determine if they correctly represent either proportional or non-proportional relationships. Students must be aware of how proportionality is represented in a variety of representations.

Understanding Tasks:
SMP 2: Reason Abstractly and Quantitatively:
The progression of understanding tasks provides students opportunities to compare and contrast key characteristics of proportional and non-proportional relationships and their representations. Students must evaluate the validity of statements and create their own organizer of how proportional relationships can be represented in a variety of models.

Interpersonal Tasks:
SMP 2: Reason Abstractly and Quantitatively:
The progression of interpersonal tasks provides students opportunities to reflect on their understanding of the mathematical characteristics of proportional and non-proportional relationships. Students reflect on their own understanding of the characteristics of each.
SMP 6: Attend to Precision:
Students must show precision in their understanding of proportional and non-proportional reasoning by reflecting on characteristics of each and how it looks in everyday life around them.

Self-Expressive Tasks:
SMP 6: Attend to Precision:
Students must show precision in their understanding of proportional and non-proportional reasoning in order to create a cinquain or write explanation of a real-world examples. Good use of vocabulary will aid them in completion of these tasks.
SMP 4: Model with Mathematics:
Students will represent their understanding of both proportional and non-proportional relationships and representations by creating a scenario they see around them along with a corresponding table, graph and equation.

Misconceptions:

This task rotation sets out to help students to debunk misconceptions that arise about proportional and non-proportional relationships. Students often think that if a relationship is linear (meaning increasing or decreasing at the same rate) that it has to be proportional. The fact is that proportionality is defined by a linear relationship that goes through the origin (graphically speaking); has an x-coordinate of 0 in one of their coordinates (in a table); is in the form $y = kx$ and later $y = mx$ (in an equation); or does not have a constant value that is added or subtracted to the unit rate (in a scenario). This misconception is confronted throughout this task rotation in practicing vocabulary through knowledge cards and applying it in their explanations throughout many of the tasks. Finally, students showcase their understanding of proportional relationships by applying their mathematical understanding of proportional relationships by to the world around them.

Implementation of the Assessment Menu:

This assessment menu is an excellent way in which to formatively assess your students. These assessment tasks could also be summative. (Note: Some tasks require a scoring guide or rubric for proper summative evaluation). However, teachers need to note that not all tasks are assigned for students to complete. Students are to select at least one task from each row (complexity level) and one task from each column (learning style). The purpose of these tasks is to push student thought in a variety of modalities. Also, student choice is key with an assessment menu. Differentiated instruction and assessment is a key outcome of implementing the assessment menu. These tasks can be assigned as bell work, class work or homework. It is important that these tasks are not to be assigned at one time. These tasks are not to be used for work that a substitute teacher assigns in a teacher's absence.

Assessment Menu—Proportional vs Non-Proportional Relationships

Standard 7.RP.A.2: Recognize and represent proportional relationships between quantities.

7. RP.A.2.A: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

7. RP.A.2.B: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Directions: Complete four tasks. You must select at least one task from each row and one task from each column.

<table>
<tr><th>Level</th><th>Mastery</th><th>Understanding</th><th>Self-Expressive</th><th>Interpersonal</th></tr>
<tr><td>1</td><td>Choose between proportional relationship or non-proportional relationship.

Create a fist list of 5 important characteristics of that relationship.</td><td>Group the following representations as proportional or non-proportional relationships. Justify your groupings. Provide the constant of proportionality if applicable.
A.
<table><tr><td>X</td><td>0</td><td>2</td><td>4</td><td>6</td></tr><tr><td>Y</td><td>0</td><td>1</td><td>2</td><td>3</td></tr></table>B.
<table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Y</td><td>5</td><td>10</td><td>15</td><td>20</td></tr></table>C.
<table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Y</td><td>0</td><td>1</td><td>4</td><td>9</td></tr></table>D.
<table><tr><td>X</td><td>1</td><td>3</td><td>8</td><td>11</td></tr><tr><td>Y</td><td>3</td><td>9</td><td>24</td><td>33</td></tr></table></td><td>Choose a value between 3 and 8 for a constant of proportionality. Represent this constant of proportionality in a graph, table and equation.

Identify if your representations demonstrate a proportional or non-proportional relationship.</td><td>Copy the following organizer and fill in the blanks with information about your knowledge of proportional relationships.

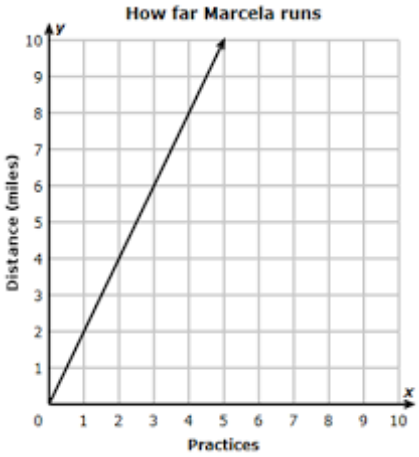
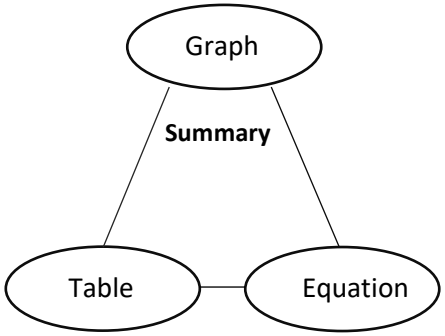
I know what I know about proportional relationships.

First, I know _____
In addition, I know _____
Finally, I know _____</td></tr>
<tr><td>2</td><td>Create two tables that have a constant of proportionality of 12. One table should exhibit a proportional relationship while the second table should exhibit a non-proportional relationship.</td><td>Identify each statement as an Always, Sometimes or Never. Justify your answer through writing or providing examples or counter-examples.

1. Proportional relationships go through the origin whereas non-proportional relationships do not go through the origin.
2. Proportional relationships have a constant of proportionality.
3. Non-proportional relationships have a constant of proportionality.</td><td>Respond to the following idea:

What if there were only proportional relationships seen in the real-world? How could life be different?</td><td>Constant of proportionality is represented multiple ways such as in graphs, equations and tables.

Choose one representation above you feel comfortable in determining the constant of proportionality. Why do you find it easy? How do you find the constant of proportionality in that representation?</td></tr>
</table>

Level	Mastery	Understanding	Self-Expressive	Interpersonal
3	Jenna was told the following representations were all proportional and had a constant of proportionality equal to 2. What's wrong with that statement? Correct any misconceptions. $y = 2x + 2$ X | 0 | 2 | 4 y | 0 | 6 | 12 	Using the three-way tie drawn below, write how the three representations help to mathematically illustrate a proportional relationship. 	Write a cinquain on either proportional or non-proportional relationships. Follow the formula below: 1st line—Title/Focus 2nd line—Two Descriptive Adjectives 3rd line—Three Action Verbs 4th line—Four-Word Phrase 5th line—One-Word Conclusion	Constant of proportionality can be found around us every day. Describe two instances in which you see constants of proportionality around you. How are they used? What kind of information do they provide for you?

Teacher Notes for Assessment Menu:

8.G.B Pythagorean Theorem

Rationale for Assessment Menu:

Pythagorean theorem is a new concept in 8th grade. Students should already know the definition of a right triangle. This assessment menu provides assessment for all three standards involving Pythagorean theorem. Students will show that they understand the proof of the Pythagorean theorem and its converse. They will apply the Pythagorean theorem to determine unknown side lengths in real-world problems. Students will use Pythagorean theorem to find the distance between two points on a coordinate system.

Assessment Menu Levels of Complexity Explained:

The assessment menu utilizes the four learning styles of mastery, understanding, interpersonal and self-expressive. Each learning style has three levels of complexity:

Level One:
The tasks allow students to identify, label and describe right triangles. It also allows them to prioritize what they have learned about Pythagorean theorem.

Level Two:
Level two activities allow students to apply the Pythagorean theorem to solve real-world and mathematical problems.

Level Three:
Students construct proofs and create problems that can be solved using Pythagorean theorem.

CCSS-M Standard(s):

8. G.B.6 - Explain a proof of the Pythagorean theorem and its converse.

8. G.B.7 - Apply the Pythagorean theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8. G.B.8 - Apply the Pythagorean theorem to find the distance between two points in a coordinate system.

Standards for Mathematical Practice (SMP):

The tasks could utilize many of the SMPs. Below are the major SMPs that each task utilizes.

Mastery Tasks:
SMP 6 Attend to Precision
These tasks provide students opportunities to show their understanding of the Pythagorean theorem. Students label diagrams using correct terminology, explain the Pythagorean theorem, and determining a missing side length.

Understanding Tasks:
SMP 1: Make Sense of Problems and Persevere in Solving Them
The tasks as hand have students making sense of the problems in order to solve. They will have to analyze the information given to identify the unknowns and properly apply the Pythagorean theorem in order to derive an answer.

Interpersonal Tasks:
SMP 4: Model with Mathematics
Students will model how to use the Pythagorean theorem by drawing diagrams and writing about real-world scenarios.

Self-Expressive Tasks:
SMP 2: Reason Abstractly and Quantitatively
These tasks will require students to show reasoning and understanding of the converse of the Pythagorean Theorem.

Misconceptions:

Some common misconceptions when teaching the Pythagorean theorem is often students cannot correctly identify the parts of a right triangle (2 legs "a" and "b" and a hypotenuse "c"). Often students believe that hypotenuse can be found in any triangle, rather than only in a right triangle. Students often forget that the hypotenuse is the longest side of a triangle which is across from the right angle.

Implementation of the Assessment Menu:

This assessment menu is an excellent way in which to formatively assess your students. These assessment tasks could also be summative. (Note: Some tasks require a scoring guide or rubric for proper summative evaluation). However, teachers need to note that not all tasks are assigned for students to complete. Students are to select at least one task from each row (complexity level) and one task from each column (learning style). The purpose of these tasks is to push student thought in a variety of modalities. Also, student choice is key with an assessment menu. Differentiated instruction and assessment is a key outcome of implementing the assessment menu. These tasks can be assigned as bell work, class work or homework. It is important that these tasks are not to be assigned at one time. These tasks are not to be used for work that a substitute teacher assigns in a teacher's absence.

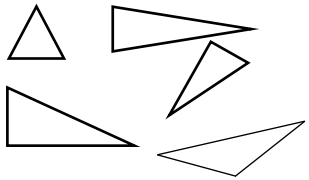
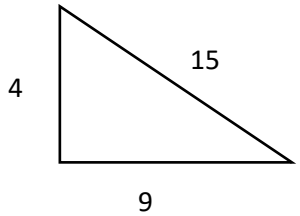
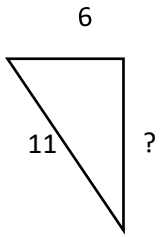
Pythagorean Theorem

STANDARD(S):

8.G.B.6: Explain a proof of the Pythagorean Theorem and its converse.

8.G.B.7: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.B.8: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Level	Mastery	Understanding	Self-Expressive	Interpersonal
1	Draw a right triangle and label the following: • 2 Legs • Hypotenuse • Right Angle	From the shapes below, identify the ones for which the Pythagorean Theorem can be applied. 	Explain in your own words the purpose of the Pythagorean theorem.	Copy the following organizer and fill in the blanks with the information about your knowledge of the Pythagorean theorem. First, I know _____ In addition, _____ Finally, I know _____
2	Explain how the Pythagorean theorem, $a^2 + b^2 = c^2$ relates to the sides of a right triangle.	Use the Pythagorean theorem to determine if this is a right triangle. 	Plot 3 points on a coordinate plane that will form a right triangle. Find the distance between each point, then find the perimeter of the triangle.	Make a model and solve: From your house, you have to walk due north to get to your friend Leon's house and due east to get to your friend Kelsey's house. It is 4 miles from your house to Kelsey's house and a straight-line distance of 5 miles from Leon's house to Kelsey's house. How far is it from Leon's house to yours?
3	Use the Pythagorean theorem to find the missing side length in each triangle. 	If a ladder leans against a 14-foot wall and the base of the ladder is 10 feet away from the building, how long is the ladder?	Use grid paper to construct a proof of the Pythagorean theorem that shows the following: $a^2 + b^2 = c^2$ AND $c^2 - b^2 = a^2$	Create a real-world scenario where the Pythagorean theorem would be used to find an unknown length or distance.